Engineering Significance of the Prowse Memorial Bridge & History of the Welded Steel Rigid-Frame Slant-Leg Highway Bridge



Prowse Memorial Bridge, Ash Street over I-93, Derry, NH, built 1962

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Introduction

The engineering origins of the Prowse Memorial Bridge can be traced to three primary disciplines of bridge development: the development of structural welding and its application to bridges; the development of laboratory methods to determine secondary stresses in indeterminate structures using elastic models; and the development of steel rigid frame bridge. All three technologies went from experimental to practical application during the decade of the 1920s. Refinement and broader application continued through the 1930s.

Structural welding methods and equipment matured during the World War II years but has continued to evolve along with the advancements in the fields of metallurgy and electrical engineering. The deformeter method of measuring elastic models when used by Prowse in the design of the Ash Street Bridge was unchanged from its original design. Although superseded by more sophisticated testing equipment today, it remains a perfectly accurate tool for the design and analysis of structures just as a measuring tape can return some of the same measurements taken with laser equipment today. Steel rigid frame bridge design and technology advanced slowly up through WWII, after which it was coupled with welding in ways that allowed the advantages of the bridge type to be more fully realized. During the 1950s and 1960s, the welded rigid frame bridge reached new levels of sophistication of design and application – the slant-leg long-span divided-highway overpass type used by Prowse for the Ash Street Bridge being a prime example.

All three technologies experienced a great leap forward due to the creative efforts of a single man or company. Credit for the practical application of welded to bridges goes to Westinghouse Electric Company engineers who designed and erected the first all-welded vehicular bridge in 1927. Credit for the rigid frame bridge goes to Arthur G. Hayden, who designed and built a form of concrete rigid frame bridge in 1923 so well-suited and ultimately widely used for highway and railroad overpasses that it came to define the type. The idea of accurately measuring the deflection of two-dimensional elastic models of bridges in order to predict the behavior of indeterminate structures such as trusses with riveted joints and rigid frames rests almost entirely with Princeton engineering Professor George E. Beggs who patented his method in 1925.

Of the three technologies, the development and application of welded bridges followed the most troubled path. The huge industry associated with riveting tirelessly fought against the acceptance of welding with warnings on the invisible and unpredictable dangers of welded connections. It took several decades and ultimately the use of x-rays to examine finished welds before welding was fully accepted by structural engineers over the simple and proven rivet or bolted connection. Even then, after WWII when welding was in such wide use for bridges in Europe and in the US as well, and the economy and structural superiority of welding was a fact, doubts continued through the 1950s and 1960s about the suitability of welded connections for certain bridge types.¹

The development of the steel rigid frame bridge did not threaten an entire industry or require a total change in the mindset of the engineering practice as a whole like electric arc welding did. It is a much narrower topic with fewer players. In a 1933 article by *Engineering News* on "Ten Years of Achievement With Rigid Frame Bridges" when the editors gave Hayden the credit for introducing the rigid frame bridge, some engineers pointed out that rigid frame bridges of different forms preceded Hayden's 1923 design by several decades. After WWII steel rigid frames and welding achieved a proper marriage and the type was refined, lightened and stretched into highly efficient and practical forms such as the slant-leg type pioneered by Robert Prowse and others.

In the case of the Beggs, he became internationally known for his invention of the deformeter as a means of predetermining the stress resistance of bridges, dams, and similar structures. Earlier researchers had studied and measured structures after their completion, but Beggs was the first to demonstrate that secondary stresses in indeterminate structures could be accurately determined in advance in the laboratory using elastic models. In 1934, at a special conference on the laboratory use of models as aids in the design and construction of structures held at Cornell University, former MIT professor Hale Sutherland called Beggs one of the five most important American contributors to structural theory. Other researchers soon followed Beggs by developing other methods and devices for measuring deflections in structural models; some were easier to use than the deformeter method, but few were as accurate.

Development of Welded Structures and Bridges

Electric arc welding using carbon electrodes was invented by Nikolay Nikolayevich Bernardos of St. Petersburg in 1881 and patented in Russia, several European countries and in the US in 1887.² Two prominent English metal working firms adopted the Bernardos system and after several years of perfecting its practical application, put it to constructive use for many types of repair work as well as the welding of tubes and fittings for boilers and piping.³ The Thompson Electric Welding Company of Boston, Mass. introduced the Bernardos electric arc welding in the US in 1888; they were apparently the first to use it commercially in the US and also the to use it for repair and pipe work.⁴

The potential structural uses of electric welding were recognized immediately. After conducting independent tests on electrically welded metal samples provided by Thompson Electric Welding Company, the editors of *Engineering News* magazine stated that

"The success of this remarkable process for welding seems now certain and its uses and novel modes of application are all the time increasing... The inventors will do the greatest service to the world if they will perfect a machine to do away with riveting, of longitudinal seams at least, by butt-welding the sheets directly to each other."⁵

During the 1890s and into the early 20th century a tremendous number of advancements in electric arc welding equipment and techniques broadened its use especially for boilers, tanks and ship hulls that due to continuous pressure often leaked or burst at their riveted seams.

In 1911, the Lincoln Electric Company, a manufacturer of electric motors in Cleveland, introduced a portable electric welding machine with variable voltage to allow a variety of metal thicknesses to be welded. Equally important was that the machine could be operated by one man, which would ultimately help make welding competitive with field riveting. Powered by gas or diesel engines, electric welding machines could now be taken directly into the field for construction and repair work. In 1917 the company started the Lincoln Electric Welding School.⁶

Through the first two decades of the 20th century, welding was used for a variety of small structural steel projects including strengthening riveted joints and reinforcing building frameworks. During World War I arc welding further advanced with the demand for ship building and repairing. The first all-welded barge was built in 1917 followed by the first all-welded ship in 1920; both were built in England.⁷

The first all-welded structural steel building was a workshop erected in Brooklyn, New York in 1920 for the Electric Welding Company of America. The one-story structure measured 40 by 134 feet and was built by the owners of the company, William Schenstrom and T.L. McBean, considered among the pioneers of structural steel welding.⁸ The Electric Welding Company of America developed a reputation as a leader in the welded construction of large steel liquid fuel storage tanks at refineries across the

country. In 1920 the company constructed an oil storage tank in Oklahoma 40 feet in diameter and 20 feet high, believed to be the largest all welded tank up to that time.⁹

The General Electric Company was also an early leader in the advancement of structural welding. In 1921 GE began a test program in conjunction with the engineering department of nearby Union College to measure the shear strength of electric welds made to join various structural steel assemblies. Following the testing GE built an all-welded truss tower, a truss crane boom and finally an elevated pedestrian through-truss bridge spanning between two factory buildings at the plant, all completed in 1921.¹⁰

One of the earliest authorities on electric welding of structures was Union College civil engineering professor Frank P. McKibben who oversaw the schools experimental and testing work for GE. McKibben had previously taught civil engineering at MIT between 1895 and 1907, then at Lehigh for another twelve years before joining the faculty of Union College in 1919. He left teaching in 1926 at the age of 55 to pursue consulting for GE on a nearly continuous basis until his death in 1936. He served as president of the American Welding Society in 1932 and wrote numerous articles on the electric welding accomplishments of General Electric.¹¹

General Electric's competitor, the Westinghouse Electric and Manufacturing Company, is usually credited with the major technical advances in structural welding that led to its use for both major buildings and bridges. In 1926 Westinghouse engineers designed and built the world's first large all-welded building, a five-story factory of heavy construction incorporating 790 tons of structural steel. It was erected at the Sharon, Pennsylvania plant by the American Bridge Company. The design followed destructive testing of full-size welded joints that radically departed from previous designs. The Sharon Building included the first welded plate girder and the first use of continuous beam framing. Although a saving of 110 tons of steel was realized over the alternative riveted design, the cost of the welding labor put the overall cost of the building somewhat higher than the cost for riveted construction. Westinghouse immediately followed the Sharon Building with the construction of six other all-welded buildings, including a factory at Derry, Pennsylvania that covered two acres. The Derry Building demonstrated for the first time an overall cost savings over riveted construction, an unsettling development for the riveting industry.

McKibben's counterpart at Westinghouse was Gilbert D. Fish, a welding and structural consulting engineer responsible for many of the early developments including the design of the first major all-welded building (Sharon Building) and the first all-welded bridges (described below). A graduate of the Columbia University engineering program, he later taught engineering at Columbia and Yale. He also served as president of the American Welding Society and wrote numerous articles on the electric welding accomplishments of Westinghouse and on the overall subject of structural welding including the first major treatise on the subject, *Arc-Welded Steel Frame Structures*, published in 1933. By 1927, General Electric was playing catch-up with Westinghouse in the area of welded structures and lured Professor McKibben away from teaching to serve as a consultant to the electric welding division. McKibben immediately began writing articles on the subject of structural welding which were almost always illustrated with photos of GE projects.

American Bridge Company also saw the potential of welded bridges early on and began its own experimental program making various types of welds to join structural steel and then testing them to failure. In 1926, James H. Edwards, assistant chief engineer of American Bridge, oversaw the welded construction of a very heavy plate girder with doubled flange plates, one slightly narrower than the other, joined along the top with fillet welds. The girder was tested to failure in the 5000-ton Olsen hydraulic compression machine at the National Bureau of Standards in Pittsburgh and the results published.¹² The test showed that the beam failed by buckling of the web plate and not by failure of any of the welds. Tests like these of full-size welded structural units convinced welding engineers and many members of the

structural engineering community that welded joints were as strong or stronger than riveted joints. Bridge engineers however, were for the most part still dragging their feet toward the new technology.

Welded Bridges

The era of welded vehicular bridges that continues today began in 1927. The first major use of electric welding on a bridge project was the double decking of a highway over the Susquehanna River at Havre de Grace, Maryland in early 1927 by the Electric Welding Company of America.¹³ The Phoenix-column through truss bridge was built in 1873 for the Pennsylvania Railroad and converted to a highway bridge shortly after the PRR built a new heavier bridge alongside it in 1908. By 1927, increased highway traffic demanded a second travel lane, which was accomplished by adding a new deck midway up the columns. The floorbeams of the new upper level deck were welded directly to the wrought iron columns using heavy gusset plates.

Shortly after the Harve de Grace bridge project started, the American Bridge Company started the first major railroad bridge strengthening project to use electric welding. The Chicago Great Western Railroad Bridge over the Missouri River at Leavenworth, Kansas consisted of two 330-foot fixed spans flanking a 440-foot swing span. The work was done between February and April 1927 without interrupting train service and involved the addition of 225,000 pounds of structural steel to the bridge by electric welding.¹⁴

Meanwhile, Westinghouse was testing forty-two full-size arc-welded structural steel joints to destruction at the Carnegie Institute as "part of the preliminary work in an organized movement to replace riveting by more economical means for connecting steel structures."¹⁵ Gilbert Fish oversaw the test program and presented his findings to the Affiliated Technical Societies of Boston on December 14, 1927:

"These tests proved that arc welded joints could be made stronger in every way than the members joined, and that arc welded plate girders assembled from plate material only could be made to resist greater bending moments than riveted plate and angle girders of the same depth and weight.¹⁶

With the test data in hand, Westinghouse Electric began construction of the world's first two all-welded bridges in late 1927. Both were railroad bridges built to carry rail spurs on their factory property. A 53-foot plate-girder bridge constructed entirely of steel plate was built at the East Pittsburgh plant to carry the Interworks Railroad between the company's Linhart works and the East Pittsburgh works. It was completed first, in December 1927, but for reasons unknown it was not credited by Westinghouse as the first all-welded bridge. It may have been because larger, heavier welded plate girders had been already successfully built for buildings and machine supports and therefore it did not represent a real advancement. Besides, Fish had a far more worthy welded bridge project nearing completion in Massachusetts.¹⁷

The distinction of "first all-welded bridge" is attributed to the 153-foot skewed through-truss span built at the Westinghouse Chicopee Falls (Mass.) plant to carry the Boston and Maine Railroad spur line over a canal and into the yard. The bridge was designed as a riveted structure by B&MRR railroad under B.W. Guppy, engineer of structures, who then agreed to let Westinghouse consulting engineer Gilbert Fish oversee the substitution of rolled shapes for built-up shapes joined entirely by electric arc welding.¹⁸ Welding saved 40 tons of steel, representing 33.5 percent of the total weight of steel in the bridge.

At the 1928 meeting of the American Institute of Electrical Engineers held in New York City in February, Albert M. Candy, Welding Engineer for Westinghouse who worked alongside Gilbert, announced his company's latest welded bridge achievements and presented a technical paper detailing the data and conclusions of the welded joint testing program.¹⁹ The *New York Times* covered the convention and

reported on Candy's address describing the many benefits of welded construction with the headline "Urges Arc Welding in Steel Buildings." Candy's comments that "the substitution of electricity for the pneumatic hammer would relieve city dwellers of the racket of riveting" was highlighted in the article.²⁰ Two electrical engineers for General Electric, Peter P. Alexander and Alexander P. Wood, also gave papers on arc welding at the meeting but were given only a passing mention in the press.

In addition to structural alterations (Havre de Grace bridge) and strengthening (Leavenworth bridge), welding was also uniquely suited to the repair structural members weakened by corrosion. Mending plates could be directly welded to the flanges of a plate girder for example, to patch a rusted-through web plate. Rusted portions of truss and chord members could be cut out and a new section welded in place. Plus the work could be done without interruption of train service. In 1928 Westinghouse demonstrated the cost-effectiveness of welded repairs on a job for the Public Service Company of New Jersey repairing an 86' pony-truss that carried a street trolley over steam railroad lines. The locomotive exhaust had severely corroded the lower chords and other members.²¹

By 1929 the business of welding bridges took off and never stopped. The first specifications for "arcwelded connections in bridges" were published in 1929, authored by Gilbert D. Fish of Westinghouse. Dozens of articles on welded structures were published in the engineering literature promoting its use (see Figure No. 1). McKibben reported that the building of electrically welded buildings and bridges increased by 50 percent over the period of a year beginning July 1928, and noted:

"The most notable advance has been in the height and size of buildings such as the new Hotel Homestead in Hot Springs, Virginia with a height of 180 feet. A second interesting development is the comparative ease by which existing bridges are strengthened by welding new steel to corroded or over stressed members, a process applied not only to highway structures but to steam railroad bridges as well. Indeed, welding could ask for no better endorsement than this adoption by some of the leading railway systems of the world." ²²



Figure No 1. Advantages of welded plate girders (McKibben 1929.)

The Electric Welding Company of America, mentioned above as one of the leaders in the field, played an active role in the development of field welding methods and techniques. A.W. Schenker, Chief Engineer for the company, developed cost effective methods for field repairs to bridges including a new method of butt-splicing.²³ The company undertook numerous projects for the railroads repairing and increasing the load capacity of bridges. Plate girder bridges were easily strengthened by welding an additional cover

plate or plates onto the old flanges using progressively narrower plates to enable the use of efficient fillet welds (see Figure no. 2).²⁴



Figure No. 2: Railroad bridge plate girder repair method. (source: *Engineering News-Record*, September 26, 1929)

An automatic welding machine for welding so-called battledeck floors – adopted from welded ship construction – was demonstrated at the 1929 convention of the American Institute of Steel Construction in Biloxi, Miss. The machine crawled along on wheels at a constant speed while a wire electrode was automatically fed from a reel.²⁵

Manual and automatic welding was soon used for the construction of solid metal floors for bridges such as the heavy concrete-filled metal trough floors used by the railroads but also for light weight applications such as metal plate battledeck or open-grate floors. Movable spans naturally benefited from lower weight welded floors. An early example is the 160-foot swing-span McKee Street Bridge between Port Arthur and Galveston Texas that was lightened in 1931 by the installation of a steel diamond-plate "battledeck" floor welded directly to the floor beams. Houston city engineer J. G. McKenzie designed and supervised the project that increased the bridge load rating from 10 to 20 tons and was done at a cost of only \$1400.²⁶

The railroads enthusiastically adopted electric welding for the repair and upgrade of their bridges because most of the work could be done without the extremely costly interruption of train service. The Erie Railroad, for example, strengthened two plate girder bridges in Akron, Ohio, in 1931 by welding additional cover plates to the girder flanges and thereby increasing the load rating from Coopers E-40 to Coopers E-73 loading.²⁷

By the early 1930s welding was also finding its place in many state highway departments for bridge repair and reinforcing projects as an economical method of increasing the load capacity. Papers discussing several successful projects were presented at the Conference on Highway Engineering held at the University of Michigan in 1933. The chief engineer of the Michigan State Highway Department, C.A. Melick, explained with detailed drawings the numerous ways in which welding was efficiently used to strengthen iron and steel truss bridges for continued service under increased loading.²⁸

But bridges constructed entirely with welded connections – referred to in the literature as "all-welded bridges" – did not immediately catch-on in the US. It was not until 1932, five years after the Westinghouse bridges, that the next all-welded bridge was completed in the US. Located in Merced, California, it was a steel viaduct, 1380 feet long, consisting of thirty-one 40-foot plate-girder spans and seven 20-foot spans.

Meanwhile, Europe's first all-welded bridge had been completed in 1930, an 88-foot camel-back ponytruss built by the Polish government over the Sludwia River on the main highway between Warsaw and Berlin. A 122-foot truss was erected in Leuk, Switzerland the same year and in 1931 a 161-foot truss was built in Plzen, Czechoslovakia.²⁹ England erected its first all-welded bridge in 1934, a five-span steelrigid frame highway bridge over railroad tracks, arguably the world's first welded rigid frame bridge (see discussion below).³⁰

In the US, attention mainly focused on all-welded plate girder bridges since they offered attractive costbenefits and simplicity of design. The elimination of separately rolled angles for the flanges and web stiffeners, the elimination of riveting labor and the reduction in loss of material section due to rivet holes resulted in significant savings in overall cost. In 1932, Samuel Eckels, Chief Engineer of the Pennsylvania State Highway Department unveiled a ten year, \$27 million program to replace 2000 bridges across the state determined unsafe for a 13-ton truck, with all-welded plate girder spans.³¹ In an effort to further the behavior of deep welded girders for long spans, the state highway departments of Missouri and Kansas teamed up in 1934 to share in the cost of fabricating and testing full-size welded plate girders to failure. The 27-foot long by 54-inch deep girders were subjected to 400 tons of loading while deflections were measured "to gain information and experience relative to the application of welding to bridge construction and also to obtain cost data under shop conditions." ³²

Cost-effective all-welded truss designs quickly followed. Ira Kelly, Bridge Designer with the Kansas State Highway Commission, published a design for an all-welded pony truss highway bridge that demonstrated a fifteen-percent overall cost savings over the same riveted design. In 1935 the largest welded truss bridge in the US and the second largest in the world was completed over the Rancocas River in Delanco, New Jersey. It was a pony truss with two fixed spans and a continuous 160-foot center swing span for an overall length of 397 feet.³³ A year later (1936) the longest welded truss bridge span in the world was completed over the Raquette River at Massena, New York as a link in the new US-to-Canada highway crossing of the St. Lawrence River. There were two main channel spans, each a high-Pratt truss of record-setting 150-foot length.³⁴

By the mid-1930s the technical literature on the subject of structural welding applied to bridges was expanding rapidly. In 1934, the American Welding Society appointed a committee to begin developing specifications for welding bridges and two years later published *Specifications for the Design, Construction, Alteration and Repair of Highway and Railroad Bridges by Fusion Welding.* The technology changed so rapidly a second edition was published in 1938 and then a third in 1941 that was renamed and "thoroughly revised and rewritten in order to bring the this work up to current theory and practice." ³⁵

The Lincoln Electric Company established the James F. Lincoln Arc Welding Foundation in 1936 to support research and development in the science, engineering and practice of structural welding. The foundation initiated a contest and awards program in 1938 for the "best technical papers on the application of arc welding to design and production."³⁶ The contests, which continue today and have resulted in the compilation of thousands of papers and designs, ultimately played an important role in the design of the Ash Street Bridge.

One of the leading writers and promoters of welded bridge construction was LaMotte Grover, Bridge Designer with the Kansas State Highway Commission. He wrote comprehensive articles on the accomplishments of European engineers who were advancing much more rapidly in the design and application of welded bridges than their US counterparts. Grover noted that the US led the world in the use of welding of mechanical equipment for industrial applications such as boilers, piping, and chemical equipment, but in the area of structural welding, particularly in bridge construction, it "has not been so outstanding."³⁷ The editors of *Engineering News* went further to say "the record of welded bridges in this

country– other than strengthening and repair jobs – is unimpressive, and when compared with that in Europe is hardly worthy of note."³⁸ The editor did offer a bit of hope, noting that the new specifications "just published" by the American Welding Society (AWS) "should mark a turning point in welded bridge development."

The AWS specifications may have been the turning point, but equally important over the next decade was the extensive research and testing of all types of structural welding.³⁹ In his 1948 paper "Welded Bridges," Grover gives an overview of the advances made in the US, Canada and Europe over the preceding twelve years. The Connecticut State Highway Department for example, under the guidance of Bridge Engineer John Willis, began a welded bridge program in 1938 that resulted in 25 bridges completed by 1949, all of which were girder or rigid frame structures.⁴⁰ Two rigid frame bridges completed in 1938 in Connecticut are early examples of the type. The overall weight reduction possible with welding versus riveting made welding the choice for lift bridge spans where considerable savings could be realized in lighter operational machinery. The Treasure Island Causeway at St. Petersburg, Florida, a double bascule highway bridge completed in 1939 is one example.⁴¹

The steel shortages that followed World War II made the materials savings afforded by welding all the more attractive and propelled welded bridges into mainstream production in the US. Technologies borrowed from the massive bridge rebuilding programs in Europe, such as composite construction, grid deck and orthotropic designs, were being more readily embraced in the US.

Determining Secondary Stresses With Elastic Models

The idea of precisely measuring the deflection of two dimensional elastic models of structures in order to predict the secondary stresses and behavior of indeterminate structures such as bridge trusses with riveted joints and rigid frames rests nearly entirely with Princeton engineering Professor George E. Beggs. Beggs introduced his idea in 1922 in a paper presented at the annual convention of the American Concrete Institute entitled "An Accurate Mechanical Solution of Statically Indeterminate Structures by Use of Paper Models and Special Gages."⁴² This was followed the next year with another paper that presented "additional evidence of the correctness of the results"⁴³ and discussed the use of more complex models formed from sheets of Celluloid, an early thermoplastic material.⁴⁴

In 1924 Beggs presented his method of measuring deflections in models to the American Society of Civil Engineers (ASCE) at their annual meeting in his discussion of a paper given by A.C. Janni on the design of multiple arch systems.⁴⁵ Janni presented a simplified method of analyzing a triple-arch open-spandrel viaduct-type bridge; Beggs made a model of the bridge in order to compare Janni's theoretical results with those obtained by measurement with the deformeter.

In 1925 Beggs patented his method and the gages that he had designed and fabricated in order to take the measurements (see Figure No. 3). The device that he patented consisted of a set of mechanical instruments for inducing and measuring deflections in models.



Figure No. 3: US Patent No. 1,551,282. August 25, 1925.

As explained by Beggs, his method was based on the "much used and well-known principle – Maxwell's Theorem of Reciprocal Deflections."⁴⁶ The theorem governs the deflections in structures that result from loads applied to a structure at a given point and direction. Deflections can be plotted using influence lines to produce a deflection diagram for a structure as shown in Figure No. 4. In 1916, Beggs was called on to plot the influence lines for the reactions for the Bessemer and Lake Erie Railroad Bridge, a three span continuous deck truss nearly 1000' feet long spanning the Allegheny River near Pittsburgh.⁴⁷ The influence lines were plotted after "calculating their ordinates by the theories of deflection and least work," the accepted method of analyzing indeterminate structures at the time. Beggs then took field measurements of the actual deflections of the bridge using a calibrated measuring pole and compared the results to those predicted by mathematical theory and found the "agreement of values to be remarkably good." ⁴⁸





The actual measuring of stresses in bridges by taking accurate measurements of the completed structure under load was first done in 1883 by Wilhelm Frankel, a professor of engineering at the Polytechnic School in Dresden. Frankel invented a strain gauge (the Frankel extensometer) that he used to measure the deflections of a riveted iron truss bridge to determine the secondary stresses. "His results produced such uneasiness among German engineers that some of them abandoned the riveted truss in favor of pin connections."⁴⁹ Frankel later invented the deflection gauge and vertical and horizontal vibration gauges and "thus created the instruments required to verify bridge engineering theories and classical structural theory."⁵⁰ Several other European engineers conducted experimental measurements on bridges to determine the actual secondary stresses in the finished structure, notably including those done in 1905 on a skew truss by Willy Gehler, a successor of Frankel at Dresden Polytechnic.⁵¹

In the United States, the first major effort to determine secondary stresses on a bridge by field measurement was undertaken by David B. Steinman in the course of the construction of the famous Hell Gate Arch Bridge in 1917 for the Pennsylvania Railroad. The magnitude of the project led Steinman to utilize the bridge as "an instrument for scientific research by conducting a series of stress measurements extending through all the different stages of erection until the structure was completed."⁵² A special strain gauge of the direct reading micrometer caliper type known as a Howard Extensometer was used to obtain the stress measurements. Steinman claimed the Hell Gate Bridge to be "probably the first structure ever built in which the true stress conditions are known from experimental determination" and that "the record, on the whole, indicates a scarcity of such observations in the past, and serves to emphasize the value and importance of undertaking more investigations of this character in order to confirm or correct the results of theoretical analysis."⁵³

Beggs realized that the great need for such empirical data on the behavior of indeterminate structures was severely limited by the great cost of conducting field measurements of bridges under construction or completed and in service. More importantly, data on the behavior of a particular design is wanted before construction, not after.

Beggs credits the genesis of his idea of using Maxwell's theorem of reciprocal deflections to obtain influence lines by observing the microscopic movements of elastic structural models to a comment made

by Steinman in reference to his Hell Gate Bridge project. Steinman noted that "every influence line is a deflection diagram" and it was from this statement that Beggs postulated "it might be possible to obtain the values for an influence line by observing certain microscopic movements of elastic structural models." ⁵⁴



Figure No. 5: Deformeter gages designed and patented by Beggs [Source: Beggs, "Discussion of Paper 1571", 1925, p 1213.]

One of the first to purchase a set of the deformeter gages was Arthur G. Hayden who went to see Beggs at his Princeton laboratory in 1925 for a personal demonstration of their use (see Figure No. 5). Hayden used the gages to analyze the innovative rigid frame bridges he was designing for the Westchester County Parks Commission. In his arguments for the practical economy of rigid frame bridges Hayden quotes the deformeter studies done by Beggs in 1923 of rigidly connected frames: "frames that are rigidly connected member to member support their loads with minimum effort... the deflection under load decreases as the rigidity of the structure increases... "the work a structure must do in supporting its load is not only divided among all members of a rigidly connected structure, but the total work to be done is less by reason of the rigidity." ⁵⁵ Hayden compared the results of a mathematical analysis of a bridge with that obtained by mechanical analysis using the "Beggs method" and noted that the mathematical methods "are probably less reliable than the physical methods of analysis."⁵⁶

Another early user of the Beggs Deformeter, also in 1925, was Joseph R. Burkey, Chief Bridge Engineer of the Ohio Department of Highways who used it to check his design of the monumental Route 20 Viaduct at Ashtabula, Ohio, a concrete open-spandrel arch viaduct with seven 135-foot arch spans. Burkey describes the unique features of the bridge and his "studies of the complex behavior of the monolithic arch and deck believed to constitute work of a pioneer nature in this field."⁵⁷ A paper model of the arch span was made and the studied using the Beggs deformeter to measure displacements in the model by applying predetermined thrust, moment or shear displacements elsewhere in the model using the gages. "By applying Maxwell's theorem of reciprocal displacements and corresponding gage displacements, influence lines were obtained for thrust, moment and shear for each of the sections tested."⁵⁸ The deformeter studies provided "proof that the deck acting in conjunction with the arch rib functions in a manner similar to a stiffing truss and provides a much greater factor of safety for the arch ribs than is considered available when the ribs are analyzed by the elastic theory."⁵⁹

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By 1927 the Beggs method had been subjected to many trial tests by university laboratories and designing offices in the US and abroad with "the consensus of opinion is that results from the model are very close to the truth. For important results the experimental error does not exceed 1 per cent."⁶⁰ Hale Sutherland, professor of engineering at MIT conducted tests on a model of a concrete arch with fixed ends using the Beggs deformeter and also found the results in near perfect agreement with the mathematical theory.

With such endorsements, the Beggs Deformeter was embraced by the engineering community and used for a wide variety of structural analyses. Besides groundbreaking work like that done by Burkey defining the accurateness of indeterminate structural theory, Beggs noted that "an engineer or architect who employs models in designing secures for himself greater freedom in the choice of unusual structural forms, which may be either more economical or more artistic."⁶¹ That was exactly the case thirty years later in the design of the Ash Street Bridge were Robert Prowse exercised freedom in conceiving the design by intuition, and then determining its structural behavior with the Beggs deformeter.⁶²

The first analysis of a suspension bridge using a model and the deformeter method was done by Beggs in 1930 in his Princeton laboratory. The 1200-foot span Mt. Hope Bridge in Rhode island, completed the year before, was chosen as the model for the initial tests because D.B. Steinman, the designer of the bridge, had compiled very complete design results using Moiseiff's deflection theory against which the deformeter deflection results could be compared. The tests not only demonstrated that suspension bridge models could be used to accurately "predetermine the stresses due to live load, wind and temperature," but also exposed errors in the conventional theory that assumed equal loads on suspenders.⁶³ Based on the successful experiments on the Mt. Hope Bridge, Beggs was called upon by the California Highway Department to work with Professor R.E. Davis of the University of California at Berkley to develop and test a model of the San Francisco to Oakland Trans-Bay suspension bridge, the first actual job application of suspension bridge model analysis.⁶⁴

Arthur Hayden asked Beggs to write a chapter entitled "Deformeter Analysis" for his book *The Rigid Frame Bridge*, published in 1931. [Refer to that six-page description for an exact description of the many careful steps involved in using the deformeter gages.]

In 1934 a special conference was organized by professors from the engineering programs of Princeton, M.I.T., Cornell, Case, Ohio State and others, on the laboratory use of models as aids in the design and construction of structures and in hydraulic studies. Held at Cornell, it was first conference on the subject of models and drew several hundred attendees from across the country. The program on structural models was divided into three subject areas: (1) consideration of the deformeter method of model analysis; (2) principles and possibilities of the photoelastic method, and (3) loaded models, including suspension bridges, dams and buildings." ⁶⁵

A reporter for *Engineering News-Record* gave this account of Professor Hale Sutherland (of Lehigh University at the time) speaking on the importance of the Beggs deformeter method:

Professor Sutherland listed five American contributors to structural theory (which he pointed out is largely the achievement of Europeans) in the persons of Squire Whipple, truss analysis, 1847; C.E. Greene, moment-area theorems, 1872; G.A. Maney, slope-deflection method, 1915; George E. Beggs, deformeter method of model analysis, 1922; and Hardy Cross, methods of column analogy and moment distribution. He paid particular tribute to professor Beggs for his development of the accurate mechanical solution of statically indeterminate structures by the use of models and gages, pointing out that the contribution to the theory and practice of stress analysis was "not surpassed by any single contribution of any other investigator.⁶⁶

George Beggs died in 1939 at the age of 56 after an illness of several weeks. He had become chairman of Princeton's Department of Civil Engineering in 1937 and had served on the faculty since 1914. Born in Ashland, Illinois, he earned a civil engineering degree, with highest honors, from Columbia University in 1910.⁶⁷

Development of the Steel Rigid Frame

The first rigid frame bridge in the United States was of reinforced concrete construction, designed in 1922 by Arthur G. Hayden, Engineer in Charge of Design for the Bronx Parkway Commission.⁶⁸ Beginning in that year until 1925, eight bridges of the type designed by Hayden were built by the Commission over the Bronx Parkway. In 1925, the Bronx Parkway Commission was dissolved and reorganized as the Westchester County (N.Y.) Park Commission, which under Hayden's direction continued the use of the rigid frame bridge in conjunction with its massive Hutchinson Parkway and Cross County Parkway construction projects. Over the next five years, Westchester County built seventy-one Hayden-designed rigid frame bridges.⁶⁹

In 1926, Hayden authored a paper *Rigid Frames in Concrete Bridge Construction*, in which he described the strength, economy and architectural merits of the design. In addition, he presented two methods of structural analysis and drawings of the arrangement of steel reinforcement utilized in several different forms of rigid frame bridges. Hayden's analytical methods were embraced by the engineering community and his designs widely copied. By 1932 over two hundred rigid frame concrete bridges had been built in the U.S. and Hayden was recognized by his peers as the father of the bridge type and its leading expert.

Hayden is also credited with the design of the first steel rigid frame bridge in the U.S., built in 1928 to carry the Bronx Parkway extension over the New York Central Railroad near Mount Pleasant, New York.⁷⁰ The Mount Pleasant Rigid Steel Frame Bridge, as it was called, was built for about \$100,000, a significant savings over the estimated \$140,000 cost of the alternative design for a simple through plate girder bridge. The 100 ft. span bridge resembled the distinctive design Hayden had developed for his reinforced concrete rigid frame bridges (see Figure No. 6). It utilized a series of parallel continuous girders with integral tapered legs that increased in depth at the knees, then tapered to the center of the span along the line of a low flat arch. The so-called "knees" were where all the bending moments were concentrated and it was this detail that was ultimately subjected to extensive testing and analysis by Hayden and others as the steel rigid frame bridge was further developed and used.

Between 1928 and 1931 Hayden built six more steel rigid frame bridges for the Westchester County parkways.⁷¹ Load deflection tests were conducted on three of the bridges and showed very small deflections thereby supporting the claim that the design possessed great rigidity.⁷²



Figure No. 6: Mount Pleasant Rigid Steel Frame Bridge, 1928, the first of its type. (Source: Hodges, 1929, p. 510).

Although Hayden originated the type of rigid frame bridge that proved economical and well suited for highway overpass structures, he was strictly not the first to intentionally utilize rigid frame principles in bridge construction. European engineers preceded Americans in the use of rigid frame bridges of both reinforced concrete and structural steel construction. Dozens of concrete rigid frame bridges were built in Europe during a ten-year period beginning in 1904.⁷³ The Stephanie Bridge, built in 1885 over the Danube Canal in Vienna, Austria, is considered one of the earliest steel rigid frame bridges.⁷⁴ It was a three span continuous girder structure with a main center span of 196 feet. The side spans were anchored with immense counterweights causing the main span to act as a "restrained girder."⁷⁵



Figure No. 7: First "Split beam arch" rigid frame bridge (Reichmann, 1936, p. 88).

In 1929 the so-called "split beam arch" was introduced, apparently by the American Bridge Company, first for a long clear span building in Chicago, then for a single-span bridge (see Figure No. 7). The first split-beam arch bridges were of riveted construction and consisted of a rolled beam span rigidly connected to I-beam legs by a curved gusset-plate knee brace. The bottom flange of the girders was cut free, bent in a curve and re-riveted to a new infill web plate with a curved soffit (see Figures Nos. 7 & 8).⁷⁶ The application of welding to this design was the next logical step and with that step the first all-welded rigid frame bridge in the US was born.



Figure No. 8: Two span "Split beam arch" rigid frame bridge fabricated and built by American Bridge Company for State of Illinois in 1936; grade separation project; to carry four tracks of the New York Central and the Chicago Rock Island and Pacific Railroad over Cermak Road, Chicago. (Source: Reichmann, 1936, p. 89).

Welded Rigid Frame Bridges

The first all-welded rigid frame bridge is believed to be a five-span steel-rigid frame highway bridge spanning the London & Northeastern Railway tracks in Newport, built in 1934 (see Figure No. 9).⁷⁷ It was of the conventional type, meaning the type of rigid frame overpass structure developed in the US by Arthur Hayden consisting of deck girders continuously joined to legs with gussets to form a knee connection. Also in 1934, an all-welded Vierendeel Truss bridge was completed in Hasselt, Belgium over the new Albert Canal – technically also a rigid frame in that the truss lacks diagonals and instead utilized rigid corner connections at each panel points to join the verticals to the chords.



Figure No. 9: (Source: Engineering News-Record, May 17, 1934, p. 626.)

The collapse of Hasselt Bridge only three years later during cold weather, was caused by brittle fracture of the steel in the area of the welds. Tests revealed that the potential for brittle fracture increased as temperature decreased, which in-turned dampened enthusiasm for welded bridge joints by states in northern climates such as New Hampshire.

In the US, the first welded rigid frame bridge was built in 1940 as part of the approach structures to Cleveland, Ohio's new Main Avenue Bridge. It was a three span structure with a 70-foot center span flanked by a 28-foot and 25-foot side spans. The leg-to-girder connections were designed on the split-arch principle in which a section of the bottom flange and adjacent web is cut from the main span girder and a curved transition section is welded in place (see Figure No. 10).⁷⁸



Figure No. 10: First welded rigid frame bridge in the US, Main Avenue Bridge, Cleveland, Ohio. (Source: Plummer, 1940, p. 56.)

The origin of the welded steel rigid frame slant-leg bridge can has been traced to two engineering professors from the University of Natal, Durban, South Africa, J.R. Daymond and M.S. Zakrzewski, who submitted a design to the Lincoln Foundation "Welded Bridges of the Future 1949 Award Program" (see Figure No. 11). Apparently the first design of the type, it was not determined if the bridge was ever constructed or what bridge was the first slant-leg built.



Figure No. 11: Probably the first design for a slant-leg rigid-frame bridge, 1949. (Source: Clark, 1950, p. 140).

A little more than a decade later, Robert Prowse would design the Ash Street Bridge, a slant-leg overpass structure that became a standard design used across the US.

In 1994, an analysis of a slant leg highway bridge was done by the engineering department at Florida International University in conjunction with the Florida Department of Transportation to study the dynamic behavior of the bridge under moving loads. According to the authors, "the slant-leg bridge is one type of the most widely used bridges highway bridges in the world... [but] unfortunately little dynamic behavior of slant-legged rigid-frame bridges has been reported yet." ⁷⁹ The need paper noted that "this type of bridge can reduce the depth of the main girder and save material in the superstructure; consequently the ratio of live load to dead load will be comparatively large... therefore the investigation of the responses of slant-legged rigid-frame bridges due to moving loads is very important and practically significant."

Notes

"http://www.lincolnelectric.com/corporate/about/history. The welding school, still in operation, has trained over 100,000 students since its inception in 1917.

⁷ McKibben, F.P. "Arc Welding of Steel Structures." *Journal of the American Welding Society*, 2 (February, 1923): 42.

⁸ Gilbert D. Fish. Arc-Welded Steel Frame Structures. (New York: McGraw-Hill Book Company, 1933): 8.

⁹ F.P McKibben. "Arc Welding of Steel Structures." *Journal of the American Welding Society*, 2 (February, 1923): 47-48.

¹⁰ McKibben, 1923, pp 32-37.

¹¹ "Frank P. McKibben." Obituary. New York Times, November 28, 1936.

¹² Whittemore, H.L. "Test of an Arc Welded Plate Girder by the American Bridge Co. and the U. S. Bureau of Standards." *Journal of the American Welding Society*, 6 (January, 1927): 42-48.

¹³ J.N. Mackall. "Doubledecking the Havre de Grace Highway Bridge." *Engineering News-Record*, 98 (June 2, 1927): 898-900.

¹⁴ American Railway Engineering and Maintenance of Way Association. "Electric Welding Reduces Cost of Strengthening Bridge." *Proceedings of the Annual Convention of the American Railway Engineering and Maintenance of Way Association*, 23 (July 1927): 279-281. See also: *Railway Age*. "Use Electric Welding Process to Strengthen Bridge." *Railway Age*, 82 (June 18, 1927): 1944-1946.

¹⁵ G.B. Fish. "Examples of Arc Welded Steel Construction." *Journal of the Boston Society of Civil Engineers*, 15 (February 1928): 59.

¹⁶ Ibid.

¹⁷ A.G. Bissell. "Plate-Girder Railway Bridge Built by Welding." *Engineering News Record*, 100 (February 23, 1928): 322-323; Fish, "Examples of Arc Welded Steel Construction," pp. 59-70. In an address before the Affiliated Technical Societies of Boston given December 14, 1927, Fish states that the East Pittsburgh plate girder bridge is completed and in service, and that the Chicopee truss bridge is under construction. In subsequent articles Fish and others state that the Chicopee truss was the first all welded bridge.

¹⁸ Gilbert D. Fish, "First Arc-Welded Railway Truss Bridge." *Engineering News-Record*, (July 26, 1928): 120-123.

¹⁹ A.M. Candy. "Arc-Welded Structures and Bridges." *Journal of the American Institute of Electrical Engineers*, 47 (April 1928): 274-277.

²⁰ "Urges Arc Welding in Steel Buildings." *New York Times*, February 18, 1928.

²¹ L.B. Woodruff. "Repairing a Truss Bridge Under Traffic by Arc Welding." *Engineering News-Record*, (October 25, 1928): 628-630. For a discussion of other notable bridge repair projects using electric welding were done in 1928 in Rochester (NY) and Pittsburgh (PA) see G. J. Green, "Repair of Corroded Bridges." *Iron Age*, 121 (March 8, 1928): 677.

¹ See Brodie, 1952, "Are We Ready for All-Welded Railroad Bridges," and Marks, 1961, "Highway Bridges...Welded or Riveted?

² Nicholas De Bernados and Stanislas Olszewski. Process of and apparatus for working metals by the direct application of electric current. US Patent 363,320, dated May 17, 1887.

³ Handy, William H. "Electric Arc Welding." *Electric Power*, 9 (January, 1896): 7-8.

⁴ "The Fibrous Structure of an Electric Weld." *Engineering News*, 22 (September 14, 1889): 260.

⁵ Ibid.

⁶ Lincoln Electric Company. "115 Years of Excellence." A history of the company available online at

²² General Electric Review. "Welded buildings increase 50 per cent in one year." *General Electric Review*, 32 (September 1929): 476; F.P. McKibben. "Structural Steel Welding." *General Electric Review*, 32 (November 1929): 622-625.

²⁵ American Welding Society. "Welding Developments in 1929." *Journal of the American Welding Society*, 9 (January, 1930): 44.

²⁶ A.F. Davis. "Texas Bridge Features Arc-Welded Battleship Deck Floor Construction."

²⁷ A.M. Knowles. "Reinforcing Main-line Railway Bridge by Welding. *Engineering News-Record*, 107 (September 10, 1931): 411-412.

²⁸ C.A. Melick, "Old Steel Road Bridges Restored by Welding." *Engineering News-Record*, 110 (June 1, 1933): 706-708.

²⁹ Grover, 1934, p. 361

³⁰ Engineering News-Record. "First All-Welded Highway Bridge in England Recently Completed. Engineering News-Record 112 (May 17, 1934): 626.

³¹ "Low Cost I-Beam Bridges for Pennsylvania Highways. *Engineering News-Record*, 110 (June 1, 1933): 709-710.
³² LaMotte Grover. "Arc-Welded Bridge Girders Tested to Failure." *Engineering News-Record*, 115 (September 19, 1935): 392.

³³ A.F. Davis. "Largest All-Welded Bridge Completed in New Jersey." *Journal of the American Welding Society*, 14 August 1935): 2-4.

³⁴ G.L. Dresser. "Welded Highway Bridge Trusses of 150-ft. Span." *Engineering News-Record*, 116 (January 9, 1936): 40-41.

³⁵ American Welding Society. *Specifications for Welded Highway and Railway Bridges. Design, Construction and Repair.* New York: American Welding Society, 1941.

³⁶ Information about the James F. Lincoln Arc Welding Foundation is available at <u>www.jflf.org</u>. The following information is from that website: The first contest in 1938 drew the submission of 1,981 papers. Of these, 446 papers were given cash awards totaling \$200,000 (\$2.68 million adjusted for inflation), with the top award being worth \$13,700 (\$183,000 in today's dollars). The program was international in scope, with the First Grand Award going to a project submitted by the president and a stockholder of Wellman Engineering Company of Cleveland, Ohio, and the Second Grand Award being won by engineers on the staff of Diagrid Structures, Ltd. of London, England.

³⁷ LaMotte Grover. "Foreign Countries Lead U.S. in Welded Bridges." *Engineering News-Record*, 116 (May 14, 1936): 703-709.

³⁸ "Behind in Bridge Welding." Engineering News-Record, 116 (May 14, 1936): 712-713.

³⁹ LaMotte Grover provides a comprehensive discussion of the testing programs and experimental advances in structural welding in his paper "Recent Trends in Concepts of Design for Welded Steel Structures." *Welding Journal*, 25 (November 1946): 1091-1108. An extensive bibliography accompanies the paper.

⁴⁰ See the bibliography for two papers by John F. Willis.

⁴¹ LaMotte Grover. "Welded Bridges." *Welding Journal*, (October, 1948): 812-826.

⁴² George E. Beggs. "An Accurate Mechanical Solution of Statically Indeterminate Structures by Use of Paper Models and Special Gages." *Proceeding of the American Concrete Institute*, 18 (1922): 58-82.

⁴³ George E. Beggs. "Design of Elastic Structures from Paper Models." *Proceeding of the American Concrete Institute*, 19 (1923): 53-66.

⁴⁴ Celluloid is a trade name registered in 1870 for a nitrocellulose thermoplastic material produced by the reaction of nitric and sulfuric acids on pure cellulose; used for billiard balls in the 19th century; guitar picks were first made from the material in 1922. see H.R. Clauser, *Encyclopedia of Engineering Materials and Processes*, 1963, p. 114; also www.wikipedia.org under "Celluloid."

⁴⁵ George E. Beggs. Discussion of Paper No. 1571, A.C. Janni, "The Design of a Multiple Arch System and Permissible Simplifications." *American Society of Civil Engineers Transactions* 88 (1925): 1208-1229.

⁴⁷ Beggs, 1922, pp. 58.

⁴⁹ Steinman, 1918, p. 1042.

 ²³ A.W. Schenker. "Á New Welded Girder Splice." *Engineering News-Record*, (November 26, 1929): 507-508.
²⁴ Engineering News Record. "Repairing railway bridge girders by welding on new cover plates." *Engineering News Record*, 103 (September 26, 1929): 506-507.

⁴⁶ Ibid., p. 1209.

⁴⁸ Beggs, 1922, p. 59.

⁵⁰ Karl-Eugen Kurrer. *The History of the Theory of Structures: from Arch Analysis to Computational Mechanics*. Berlin: Ernst & Sohn, 2008, p. 731.

⁵¹ D.B. Steinman. "Stress Measurements on the Hell Gate Arch Bridge." *American Society of Civil Engineers Transactions*, 82 (1918): 1040.

⁵² Ibid.

⁵³ Steinman, 1917, pp. 1041, 1047-1049.

⁵⁴ Beggs, Discussion of Paper No. 1571, p. 1209.

⁵⁵ Hayden, Arthur G. "Rigid Frames in Concrete Bridge Construction." Engineering News-Record, 96 (April 29, 1926): 686.

⁵⁶ Ibid., p. 689.

⁵⁷ Burkey, J.R. "Features of an Open-Spandrel Arch Viaduct." Engineering News Record, 101 (December 20, 1928): 919.

⁵⁸ Ibid., p. 922.

⁵⁹ Ibid.

⁶⁰ George E. Beggs. "The Use of Models in the Solution of Indeterminate Structures." *Franklin Institute Journal*, 203 (March, 1927): 384.

⁶¹ Ibid., p. 376.

⁶² Robert Prowse describes his use of the deformeter in "All-Welded Frame Type Stringer Span Design Proved by Model Tests," Modern Welded Structures, Vol. 1 (Cleveland, Ohio: The James F. Lincoln Arc Welding Foundation, 1963), pp. 103-105. See also Garvin, James L. "Ash Street Bridge." NHDHR Individual Inventory Form # LON0116 for a complete discussion of Prowse's design work related to the bridge.

⁶³ George E. Beggs, Elmer K. Timby and Blair Birdsall. "Suspension Bridge Stresses Determined by Model." *Engineering News-Record*, 108 (June 9, 1932): 828.

⁶⁴ Ibid.

⁶⁵ The conference was held in conjunction with the annual meeting of the Society for the Promotion of Engineering Education. See, "Models as Aids in Design and Construction." *Engineering News-Record*, 112 (June 28, 1934): 843-846.

⁶⁶ Engineering News-Record. "Models as Aids in Design and Construction." Engineering News-Record, 112 (June 28, 1934): 843-846.

⁶⁷ "Professor G.E. Beggs, 56, Dies." New York Times, November 24, 1939.

⁶⁸ Arthur G. Hayden. "Continuous Frame Design Used for Concrete Highway Bridges." *Engineering News-Record*, 90 (January 11, 1923): 73-75. Hayden had commissioned Princeton University engineering laboratory to conduct tests on models of reinforced concrete knees equipped with strain gauges that were then tested to failure. It is likely that Hayden met Princeton engineering Professor George E. Beggs at that time, who would later conduct deformeter measurement on models of Hayden's bridge designs. See: Arthur G. Hayden, "Tests of Knees for Continuous Frame Bridges." *Engineering News-Record*, 90 (January 18, 1923): 108-110.

⁶⁹ Engineering News-Record, 1933: 531-533.

⁷⁰ R.M. Hodges. "Rigid-Frame Construction Applied to Structural Steel." *Engineering News Record*, 102 (March 1929): 509-512.

⁷¹ L.G. Holleran. "Rigid-Frame Bridges in Westchester County." *Civil Engineering*, 2 (October, 1932): 652.

⁷² R.H. Hodges. "Deflection Tests Show Rigidity of Steel Rigid-Frame Bridges." *Engineering News-Record*, 107 (September 3, 1931): 371.

⁷³ A.A. Brielmaier. "Early Rigid Frame Bridges." *Civil Engineering*, 2 (October, 1932): 653; L.J. Mensch. "Early Use of Rigid Frame Bridges." *Civil Engineering*, 5 (October 1935): 642; L.J. Mensch. *Re-Inforced Concrete Constructions*. Chicago: Cement & Engineering News, 1904.

⁷⁴ L. J. Mensch. "Early Use of Rigid Frame Bridges." *Civil Engineering*, 5 (October 1935): 642.

⁷⁵ "Stephanie Bridge over the Danube Canal in Vienna," *Minutes of the Proceedings of the Institution of Civil Engineers*, 92 (1888): 431-432.

⁷⁶ A. Reichmann. "Steel Rigid Frame for Bridges and Buildings." *Journal of the Western Society of Engineers*, 41 (April 1936): 85-92.

⁷⁷ "First All-Welded Highway Bridge in England Recently Completed. *Engineering News-Record* 112 (May 17, 1934): 626.

⁷⁸ F.L. Plummer. "Welded Rigid Frames, European Style." *Engineering News-Record*, 125 (July 18, 1940): 91-93.
⁷⁹ Ton-Lo Wang, et.al. "Dynamic Behavior of Slant-Legged Rigid-Frame Highway Bridge." *Journal of Structural Engineering*, 120 (March 1994): 885.